GOALS, CONSTRAINTS, AND TRANSPARENTLY FAIR ASSIGNMENTS: A FIELD STUDY OF RANDOMIZATION DESIGN IN THE UEFA CHAMPIONS LEAGUE

ONLINE APPENDIX

MARTA BOCZOŃ AND ALISTAIR J. WILSON

Contents

Date: March 11, 2022.

Authors: Boczoń: Department of Economics, Copenhagen Business School, Porcelænshaven 16A, 2000 Frederiksberg. E-mail: mbo.eco@cbs.dk. Wilson: Department of Economics, University of Pittsburgh, 230 South Bouquet Street, Wesley W. Posvar Hall, Pittsburgh PA, 15213. E-mail: alistair@pitt.edu. For helpful comments and feedback, we thank Eric Budish, Yeon-Koo Che, Dries Gossenns, Ignacio Palacios-Huerta, Fuhito Kojima, Laszl Cst, Jean-Franois Richard, M. Utku Ünver, Leeat Yariv, and the participants of the Conference on Economic Design of SED and the International Conference on Economics of Football. Moreover, we are very grateful to the three anonymous referees for their insightful comments and helpful suggestions.

Appendix A. Proofs

A.1. **Proposition 1.** For any matching $V \in \Gamma$, each of the $K!$ possible permutations of V has strictly positive probability, so $\Pr\{V\} = \sum_{\mathbf{v} \in \mathcal{P}(V)} \Pr\{\mathbf{v}\}$, which can be rewritten using the chain rule as

$$\Pr\{V\} = \sum_{\mathbf{v} \in \mathcal{P}(V)} \prod_{k=1}^{K} \left(\Pr\{R_k \,|\mathbf{v}_{k-1}\} \cdot \Pr\{W_k \,|R_k, \mathbf{v}_{k-1}\} \right),$$

where $\mathbf{v}_{k-1}$ denotes the matches selected in steps 1 through $k-1$. Since the randomization is uniform at each step, $\Pr\{R_k \,|\mathbf{v}_{k-1}\}$ simplifies to $\frac{1}{K-k+1}$ and $\Pr\{W_k \,|R_k, \mathbf{v}_{k-1}\} = \frac{1}{\mathcal{W}_k(\mathbf{v})}$.

A.2. **Proposition 2.** The constrained $\mathcal{R}$-first element-uniform draw is distinct from: (i) a uniform draw over Γ; (ii) A sequential uniform draw of Γ-feasible team pairs; and (iii) The same draw where we switch the labeling of $\mathcal{R}$ and $\mathcal{W}$ (the Γ-constrained element-uniform draw where we draw from $\mathcal{W}$ first).

Part (i). Consider a problem of matching $\mathcal{R} = \{a, b, c, d\}$ to $\mathcal{W} = \{e, f, g, h\}$ under the constraints $H = \{ae, bf, cg, dh, ah, bg, dg\}$.[1] The three resulting feasible matchings are given by $V_1 = \{ag, be, ch, df\}$, $V_2 = \{ag, bh, ce, df\}$, and $V_3 = \{ag, bh, cf, de\}$. Under a uniform draw over $\Gamma = \{V_1, V_2, V_3\}$, the probability of V_1 is equal to $\frac{1}{3}$. However, under the constrained $\mathcal{R}$-first element-uniform draw, (and knowing that a is degenerate) the probability of V_1 is given by:

$$\Pr\{V_1\} = \Pr\{be \in V^\star\} = \sum_{x \in \mathcal{W}} \Pr\{w_1 = x\} \cdot \Pr\{be \in V^\star \,|w_1 = x\} = \tfrac{13}{36}.$$

Hence, the two procedures are distinct.[2]

Part (ii). Consider a problem of matching $\mathcal{W} = \{a, b, c, d\}$ to $\mathcal{R} = \{e, f, g, h\}$ under the constraints $H = \{bf, cg, ch, bg, dh\}$. Among the five resulting feasible matchings, only $V_1 = \{af, bh, ce, dg\}$ contains the match af. Under a sequential uniform draw of Γ-feasible team pairs $\Pr\{V_1\} = \Pr\{af \in V^\star\} = \frac{59}{308}$; However, under the constrained $\mathcal{R}$-first element-uniform draw $\Pr\{V_1\} = \Pr\{af \in V^\star\} = \frac{55}{288}$. Hence, the two procedures are distinct.

Part (iii). Consider a problem of matching $\mathcal{R} = \{a, b, c, d\}$ to $\mathcal{W} = \{e, f, g, h\}$ under the constraints $H = \{bf, cg, ch, bg, dh\}$. Among the five resulting feasible matchings, only $V_1 =$

[1] The proof requires at least a 4×4 market, as a 3×3 is degenerate with the standard symmetric group constraints and a single asymmetric same-nation exclusion. The proof can obviously be extended to any $n \times n$ market by making the remaining $(n-3)$ match partners unique through exclusions.

[2] The proof becomes more cumbersome but still goes through if we removed the dg exclusion that forces ag to be degenerate.

$\{af, bh, ce, dg\}$ contains the match af.[3] Under the constrained $\mathcal{W}$-first element-uniform draw $\Pr\{V_1\} = \Pr\{af \in V^\star\} = \frac{161}{864}$; However, under the constrained $\mathcal{R}$-first element-uniform draw $\Pr\{V_1\} = \Pr\{af \in V^\star\} = \frac{55}{288}$.[4] Hence, the two procedures are distinct.

A.3. **Proposition 3.** First, notice that any assignment V can be rewritten as a matrix $\mathbf{X}(V) \in \{0,1\}^{K\times K}$ with a generic entry $x_{ij}(V) = \mathbf{1}\{r_i w_j \in V\}$ indicating whether or not runner-up r_i is matched to winner w_j. Since V represents a perfect matching between $\mathcal{R}$ and $\mathcal{W}$, $\mathbf{X}(V)$ is a rook-matrix where each row and column have exactly one unit-valued entry with all other entries equal to zero.

Second, for any random draw over Γ_H, the expected assignment matrix is defined as $\mathbf{A} := \mathbb{E}\mathbf{X}(V) = \sum_{V\in\Gamma_H} \Pr\{V\}\cdot\mathbf{X}(V)$, with the generic entry a_{ij} representing the probability of the $r_i w_j$ match.

Next, recall that an expected assignment matrix $\mathbf{A}$ in our setting satisfies the matching constraints if:

(1) Each entry a_{ij} can be interpreted as the probability of $r_i w_j$ being part of V ($\forall i, j : 0 \leq a_{ij} \leq 1$);
(2) Excluded entries have zero probability ($\forall r_i w_j \in H : a_{ij} = 0$);
(3) Each row and column can be interpreted as the marginal probability distribution for the respective team ($\forall i, j : \sum_{k=1}^{K} a_{kj} = \sum_{k=1}^{K} a_{ik} = 1$).

As such, the matching constraints can be grouped into two distinct sets: (i) the union of the singleton constraints and the K row constraints; and (ii) the K column constraints and the result follows.[5] Consequently, the matching constraints satisfy the bihierarchy condition in Budish et al. (2013, Theorem 1) and the result follows.

A.4. **Proposition 4: Simulation Error Result.**

Proposition 4. *Simulating the mechanism $N = 10^6$ times leads to 95 percent confidence intervals smaller than ± 0.001.*

Proof. Assignments are independent draws from a fixed distribution with a probability of selecting assignment V given by $f(V)$. The probability that the particular match ab is selected is given by $p_{ab} = \sum_{V\in M(ab)} f(V)$ where $M_{ab} := \{V \in \Gamma \,|ab \in \mu\}$ is the set of matchings

[3] A similar counterexample can be constructed with the standard group restriction enforced, but would require a 5×5 market; We omit it for tractability and instead, focus on a 4×4 sub-market.

[4] The four other matchings are: $V_2 = \{ae, bh, cf, dg\}$; $V_3 = \{ag, bh, ce, df\}$; $V_4 = \{ag, bh, cf, de\}$; $V_5 = \{ah, be, cf, dg\}$.

[5] The quotas for each element a_{ij} are a min and a max of zero for the excluded singletons; a min of zero and a max of one for the non-excluded singletons; a min and a max of one for the row sum; and a min and a max of one for the column sum.

which include ab. We simulate the vector 8^2-vector $\hat{\boldsymbol{p}}$ where each element in $\hat{p}_{ab}$ is calculated from the N independent simulation assignments $\left(\hat{V}_i\right)_{i=1}^{N}$

$$\hat{p}_{ab} := \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}\left\{ab \in \hat{V}_i\right\}.$$

The vector $\hat{\boldsymbol{p}}$ has the obvious property that $\mathbb{E}(\hat{\boldsymbol{p}}) = \boldsymbol{p}$. We can use the central-limit theorem to show that $\sqrt{n}(\hat{\boldsymbol{p}} - \boldsymbol{p}) \xrightarrow{D} \mathcal{N}_{64}(\mathbf{0}, \boldsymbol{\Omega})$ where the variance-covariance matrix $\boldsymbol{\Omega}$ has a generic element given by:

$$\omega_{ab,cd} = \Pr\{ab \wedge cd\} - \Pr\{ab\}\Pr\{cd\},$$

which can be estimated by

$$\hat{\omega}_{ab,cd} = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}\left\{ab, cd \in \hat{V}_i\right\} - \hat{p}_{ab}\hat{p}_{cd}.$$

However, given a simulation-size of $N = 10^6$, a conservative estimates (as $\omega_{ab,ab} \leq \frac{1}{4}$) for the 95 percent confidence interval for each probability p_{ab} is given by $\hat{p}_{ab} \pm \frac{1.96}{2000} \approx \hat{p}_{ab} \pm 0.001$. $\square$

Appendix B. Data and Estimation of Game-Outcome Model

In order to account for variation in teams' ability while examining potential effects driven by the tournament's constraints, we estimate a commonly used structural model for football-game outcomes: the bivariate Poisson (Maher, 1982; Dixon and Coles, 1997).

B.1. **Model.** Let S_i and S_j be the random variables indicating the number of goals scored by home-team i and guest-team j in a given game. In a bivariate Poisson model with parameters $(\lambda_1, \lambda_2, \lambda_3)$ the realized scoreline (s_i, s_j) has a joint probability distribution given by

$$\Pr_{(S_i,S_j)}(s_i, s_j) = \exp\{-(\lambda_1 + \lambda_2 + \lambda_3)\} \frac{\lambda_1^{s_i}}{s_i!} \frac{\lambda_2^{s_j}}{s_j!} \sum_{k=0}^{\min(s_i,s_j)} \binom{s_i}{k} \binom{s_j}{k} k! \left(\frac{\lambda_3}{\lambda_1 \lambda_2}\right)^k,$$

where $\mathbb{E}[S_i] = \lambda_1 + \lambda_3$, $\mathbb{E}[S_j] = \lambda_2 + \lambda_3$ and $\mathbf{Cov}(S_i, S_j) = \lambda_3$.

In our specification, we follow Karlis and Ntzoufras (2003) and assume that $\ln \lambda_1 = \mu^t + \eta^t + \alpha_i^t - \delta_j^t$, $\ln \lambda_2 = \mu^t + \alpha_i^t - \delta_j^t$, and $\lambda_3 = \rho^t$., where α_k^t and δ_k^t measure the idiosyncratic offensive and defensive abilities for team k in season t (larger values indicating greater ability), μ^t denotes the season-specific constant, and η^t is the season-specific home-advantage parameter.

B.2. **Estimation.** We estimate the above model via constrained maximum likelihood separately for each season t between 2004 and 2022. For scale identification we impose two sum-to-zero constraints in each season, forcing $\sum_k \alpha_k^t = \sum_k \delta_k^t = 0$. For estimation in season t we rely on game-level data from the group stage in season t and the group and knock-out stages (except for the final game which is played on a neutral soil) in seasons $t-1$ and $t-2$. This results in a total of 408 game-level observations used in the estimation for the 2004 season, 376 observations for the 2005 season, and 348 observations for each season between 2006 and 2022. The differences in the number of observations across years result from a change to the tournament design in the 2004 season, where a second group stage feeding into the quarter-finals was replaced by the R16. Table B.1 below provides summary statistics for the UCL game-level outcomes in all seasons between 2002 and 2022.[6]

In Figure B.1 we graph the estimated parameters (defense on the horizontal axis, offense on the vertical) for the subset of teams reaching the R16 in the 2004–19 seasons, dropping those that fail to get past the group stage. The strongest teams have large positive values for both the offense and defense parameters; see for example Manchester United in 2010 and FC Barcelona in 2011. Conversely, low-performing teams have either a negative value for

[6]The 2020 season was interrupted by the outbreak of the COVID-19 pandemic. Once the seasons resumed the quarter- and semi-finals followed a one-leg format and were played on a neutral ground. For the estimation of the model parameters in the 2021-2022 seasons we assumed that the 2020 QFs and SFs were played twice, once at home, once away, each time with the same realized outcome.

Table B.1. Summary statistics for the number of goals scored in the UCL

Season	# Games	Average		Std. Dev.	
		Home	Away	Home	Away
2002	156	1.69	0.95	1.18	0.90
2003	156	1.58	1.19	1.36	1.07
2004	124	1.52	0.94	1.37	0.99
2005	124	1.69	0.97	1.46	1.07
2006	124	1.39	0.86	1.31	0.97
2007	124	1.47	1.00	1.29	1.05
2008	124	1.57	1.07	1.42	1.02
2009	124	1.45	1.19	1.34	1.28
2010	124	1.42	1.15	1.23	1.13
2011	124	1.64	1.19	1.44	1.27
2012	124	1.68	1.09	1.54	1.13
2013	124	1.63	1.31	1.30	1.10
2014	124	1.62	1.26	1.35	1.33
2015	124	1.70	1.19	1.60	1.33
2016	124	1.68	1.12	1.42	1.13
2017	124	1.84	1.19	1.68	1.23
2018	124	1.77	1.43	1.53	1.40
2019	124	1.71	1.23	1.43	1.20
2020	124	1.77	1.54	1.50	1.45
2021	124	1.48	1.48	1.25	1.33
2022	96	1.71	1.39	1.39	1.42

From the group stage onward except for the final game played on a neutral ground. At the time of writing, the 2022 season has not yet concluded and thus, the summary statistics for the number of goals in that season refer only the 96 group-stage games

the offense parameter (AS Monaco in 2015), the defense parameter (AC Milan in 2011), or both (Rangers in 2006). The large mass of teams are low-to-medium-strength with small but positive values over both the offense and defense parameters (see Table B.2 below).

Complementing the figure, Table B.3 presents summary statistics for the estimated offense and defense parameters broken out by the realized stage reached within the tournament. We find that the eight teams that progress to the quarter-finals are stronger both offensively and defensively than those eliminated in the R16. Similarly, the four teams advancing to the semi-finals have better offensive and defensive performance relative to those knocked out in the quarter-finals. A two-sample Kolmogorov-Smirnov test confirms the pattern, indicating that the empirical distributions of the offense and defense parameters in the R16 and the semi-finals are statistically different ($p = 0.002$ for offense, $p < 0.001$ for defense).

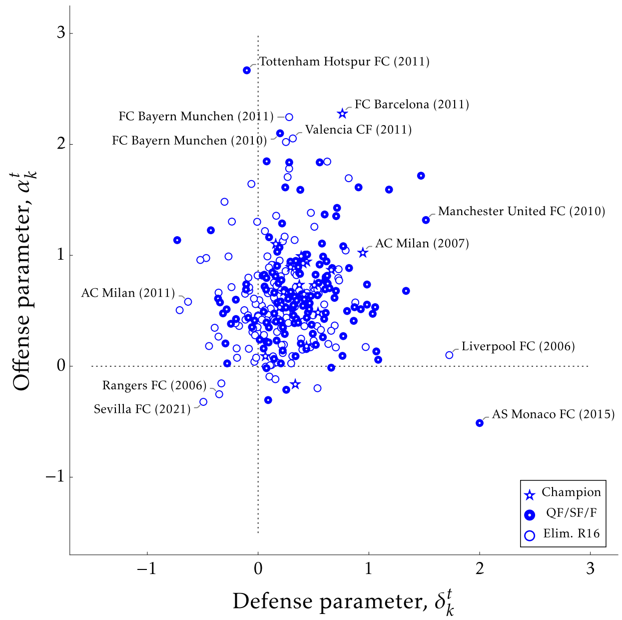


FIGURE B.1. Estimated offense and defense parameters for the R16 teams

Figure details: Figure excludes estimated parameter pairs for teams that failed to reach the R16.

TABLE B.2. Estimated coefficients of the bivariate Poisson model

Season	$\boldsymbol{\mu^t}$	$\boldsymbol{\eta^t}$	$\boldsymbol{\rho^t}$
2004	-0.25	0.42	-2.06
2005	-0.47	0.46	-2.00
2006	-0.71	0.57	-1.88
2007	-0.86	0.55	-1.99
2008	-0.49	0.46	-2.02
2009	-0.85	0.37	-1.71
2010	-1.34	0.29	-1.37
2011	-1.51	0.31	-1.33
2012	-0.69	0.27	-10.34
2013	-0.25	0.34	-2.23
2014	-0.05	0.31	-13.76
2015	-0.04	0.27	-16.38
2016	-0.09	0.34	-15.78
2017	-0.07	0.39	-14.96
2018	-0.05	0.37	-16.59
2019	-0.22	0.32	-16.06
2020	0.11	0.23	-16.71
2021	0.17	0.17	-16.46
2022	0.16	0.11	-14.73

μ^t denotes the constant term, η^t the home-effect parameter, and ρ the correlation coefficient between the number of goals scored by two opposing teams

Table B.3. Summary statistics for estimated parameters (R16 teams)

Stage	**Offense parameter**, α_k^t				**Defense parameter**, δ_k^t			
	Mean	Med.	Min	Max	Mean	Med.	Min	Max
Elim. in R16	0.59	0.50	-0.32	2.25	0.25	0.17	-0.71	10.99
Elim. in QF	0.66	0.62	-0.51	2.67	0.33	0.29	-0.73	2.00
Reach SF	0.74	0.69	-0.43	2.28	0.61	0.43	-0.43	11.52

For 2004–2021 seasons only

Appendix C. Additional Tables and Figures

Table C.1. Format of the post-qualifying stages of the UCL

Season(s)	1st knockout phase		Group stage		2nd knockout phase			
	K1	K2	G1	G2	R16	QF	SF	F
1956-1966	16					8	4	2
1967	30	16				8	4	2
1968–1991	32	16				8	4	2
1992–1993	32	16	8					2
1994	32	16	8				4	2
1995–1997			16			8	4	2
1998–1999			24			8	4	2
2000–2003			32	16		8	4	2
2004–2022			32		16	8	4	2

K1 and K2 denote the number of teams competing in the 1st and 2nd knock-out round; G1 and G2 in the 1st and the 2nd round of the group stage; R16 in the R16, QF in the quarter-final, SF in the semi-final, and F in the final game. *Source*: Table compiled based on data provided by UEFA on https://www.uefa.com/uefachampionsleague

Table C.2. Number of same-nation exclusions in the UCL R16

Season	TOP5						POR	RUS	Total
	ENG	ESP	FRA	GER	ITA	Total			
2004		3				3			3
2005	4			2		6			6
2006	1	2				3			3
2007		2	1		2	5			5
2008	4				2	6			6
2009	4	3			2	9	1		10
2010			1		2	3			3
2011	3	2				5			5
2012				1	2	3			3
2013	1	4			1	6			6
2014	4			4		8			8
2015	2		1	4		7			7
2016	2					2		(1)	3
2017	2	4	1	2		9			9
2018	4	2			1	7			7
2019	3	2	1	2	1	9			9
2020	4	4	1	2	2	13			13
2021		3		4	2	9			9
2022	3	2	1		1	7			7
Mean	2.8	2.7	1.0	2.5	1.6	5.7			5.8

(1) indicates a constraint generated by FC Zenit (RUS) and FC Dynamo Kyiv (UKR). *Source*: Table compiled based on data provided by UEFA on https://www.uefa.com/uefachampionsleague

Table C.3. Linear regression for simulation results

Regressor	Estimate	Std.Err
Assignment's dimension	-1.33E-02	6.50E-05
Number of constraints	2.73E-03	8.90E-06
Number of valid assignments	2.44E-06	2.21E-08
Independent constraints=Yes	9.02E-02	4.03E-04
Negatively-correlated constraints=Yes	8.40E-02	4.11E-04
Positively-correlated constraints=Yes	9.43E-02	4.12E-04

The dependent variable is the efficiency loss ϕ. The number of observations is equal to 540,000

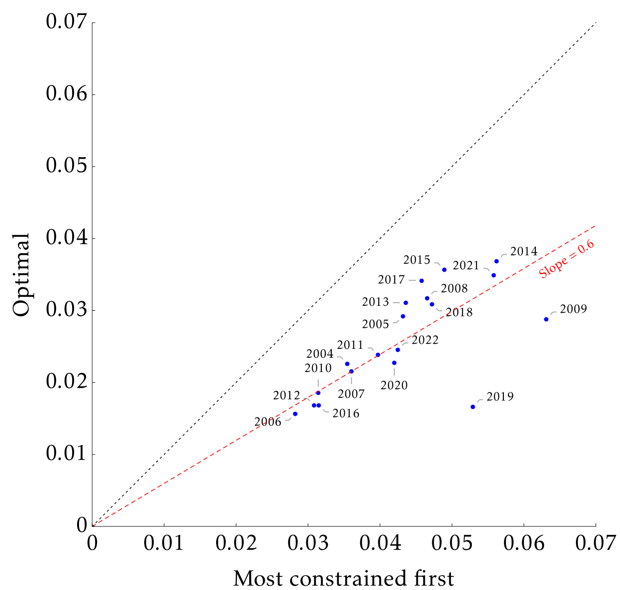


Figure C.1. Comparison of fairness distortion measure Q: Procedure that matches most-constrained teams first versus current UEFA procedure

Figure details: Red dashed line indicates fitted linear relationship.

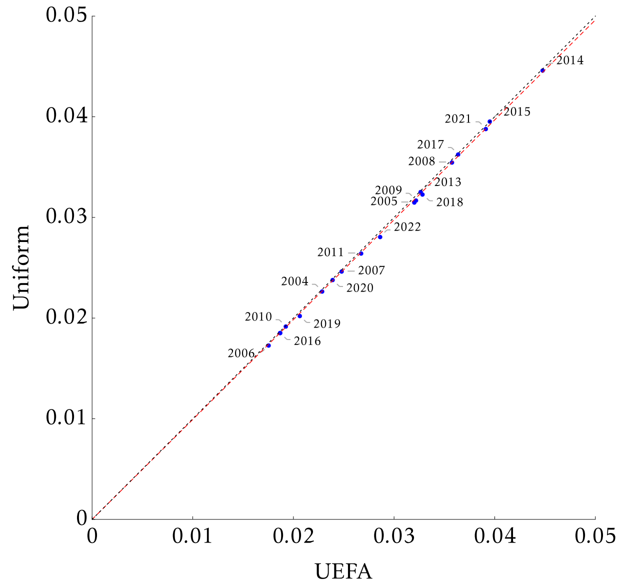


FIGURE C.2. Comparison of fairness distortion measure Q: Uniform draw over Γ versus current UEFA procedure

Figure details: Red dashed line indicates fitted linear relationship.

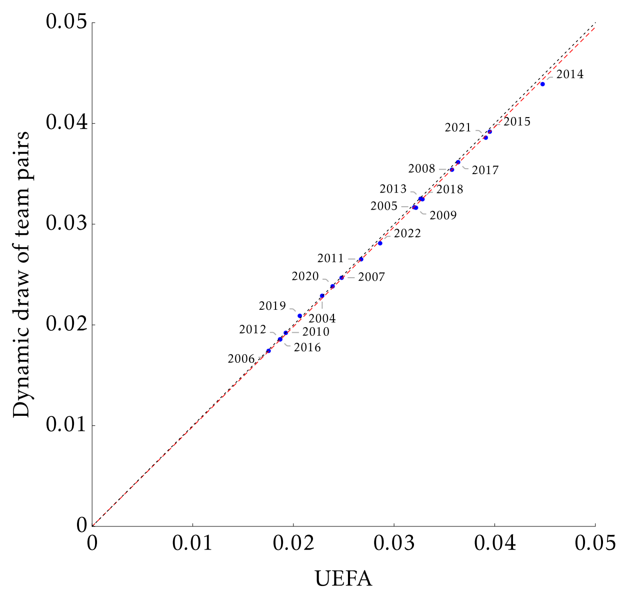


FIGURE C.3. Comparison of fairness distortion measure Q: Sequential uniform draw of Γ-feasible team pairs versus current UEFA procedure

Figure details: Red dashed line indicates fitted linear relationship.

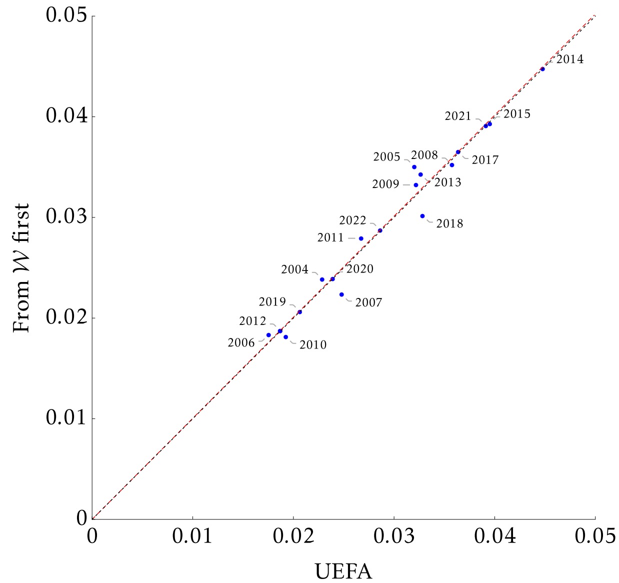


FIGURE C.4. Comparison of fairness distortion measure Q: Γ-constrained element-uniform draw where we draw from $\mathcal{W}$ first versus current UEFA procedure

Figure details: Red dashed line indicates fitted linear relationship.

Appendix D. Expected assignment matrices for the UCL seasons 2004–2019

Table D.1. Season 2004

	FC Bayern Munchen	FC Lokomotiv Moskva	RC Deportivo La Coruna	Real Sociedad de Futbol	VfB Stuttgart	FC Porto	AC Sparta Praha	RC Celta de Vigo
Olympique Lyonnais	0.	0.123	0.164	0.164	0.123	0.139	0.123	0.164
Arsenal FC	0.123	0.	0.164	0.164	0.123	0.139	0.122	0.164
AS Monaco FC	0.127	0.127	0.	0.173	0.127	0.147	0.127	0.172
Juventus	0.127	0.127	0.172	0.	0.128	0.147	0.127	0.173
Manchester United FC	0.123	0.123	0.164	0.164	0.	0.14	0.124	0.164
Real Madrid CF	0.25	0.25	0.	0.	0.25	0.	0.25	0.
Chelsea FC	0.123	0.124	0.164	0.163	0.123	0.14	0.	0.164
AC Milan	0.127	0.127	0.173	0.172	0.127	0.147	0.127	0.

Table D.2. Season 2005

	Liverpool FC	Real Madrid CF	FC Bayern Munchen	Manchester United FC	PSV Eindhoven	FC Barcelona	SV Werder Bremen	FC Porto
AS Monaco FC	0.	0.126	0.145	0.198	0.132	0.122	0.146	0.131
Bayer 04 Leverkusen	0.251	0.	0.	0.251	0.169	0.16	0.	0.17
Juventus	0.185	0.124	0.	0.184	0.123	0.117	0.142	0.124
Olympique Lyonnais	0.198	0.128	0.145	0.	0.131	0.122	0.145	0.131
Arsenal FC	0.	0.19	0.214	0.	0.	0.182	0.215	0.2
AC Milan	0.181	0.12	0.138	0.181	0.121	0.	0.138	0.121
FC Internazionale Milano	0.185	0.123	0.142	0.185	0.124	0.117	0.	0.124
Chelsea FC	0.	0.189	0.215	0.	0.2	0.181	0.215	0.

Table D.3. Season 2006

	FC Bayern Munchen	AFC Ajax	SV Werder Bremen	SL Benfica	PSV Eindhoven	Real Madrid CF	Chelsea FC	Rangers FC
Juventus	0.	0.131	0.133	0.132	0.129	0.192	0.154	0.128
Arsenal FC	0.153	0.	0.157	0.158	0.153	0.226	0.	0.153
FC Barcelona	0.16	0.164	0.	0.168	0.161	0.	0.188	0.16
Villarreal CF	0.16	0.164	0.167	0.	0.16	0.	0.19	0.16
AC Milan	0.129	0.132	0.133	0.132	0.	0.192	0.152	0.13
Olympique Lyonnais	0.137	0.14	0.143	0.143	0.137	0.	0.162	0.137
Liverpool FC	0.132	0.137	0.135	0.135	0.132	0.197	0.	0.132
FC Internazionale Milano	0.129	0.132	0.133	0.132	0.129	0.193	0.153	0.

Table D.4. Season 2007

	FC Barcelona	FC Internazionale Milano	PSV Eindhoven	AS Roma	Real Madrid CF	Celtic FC	FC Porto	LOSC Lille
Chelsea FC	0.	0.152	0.124	0.164	0.158	0.124	0.123	0.156
FC Bayern Munchen	0.152	0.	0.124	0.164	0.155	0.124	0.124	0.157
Liverpool FC	0.148	0.148	0.	0.159	0.151	0.121	0.121	0.152
Valencia CF	0.	0.223	0.182	0.	0.	0.182	0.182	0.231
Olympique Lyonnais	0.181	0.181	0.147	0.197	0.	0.146	0.147	0.
Manchester United FC	0.148	0.148	0.121	0.158	0.151	0.	0.121	0.152
Arsenal FC	0.148	0.148	0.12	0.158	0.152	0.121	0.	0.152
AC Milan	0.222	0.	0.182	0.	0.232	0.183	0.182	0.

Table D.5. Season 2008

	Liverpool FC	FC Schalke 04	Olympiacos FC	Celtic FC	Olympique Lyonnais	AS Roma	Fenerbahce SK	Arsenal FC
FC Porto	0.	0.129	0.121	0.123	0.12	0.187	0.123	0.198
Chelsea FC	0.	0.	0.179	0.182	0.179	0.278	0.182	0.
Real Madrid CF	0.18	0.119	0.	0.116	0.114	0.174	0.116	0.181
AC Milan	0.221	0.144	0.137	0.	0.137	0.	0.141	0.22
FC Barcelona	0.181	0.12	0.114	0.116	0.	0.174	0.116	0.18
Manchester United FC	0.	0.214	0.194	0.2	0.194	0.	0.199	0.
FC Internazionale Milano	0.22	0.145	0.137	0.141	0.137	0.	0.	0.22
Sevilla FC	0.198	0.13	0.12	0.123	0.12	0.187	0.123	0.

Table D.6. Season 2009

	Chelsea FC	FC Internazionale Milano	Sporting Clube de Portugal	Club Atletico de Madrid	Villarreal CF	Olympique Lyonnais	Arsenal FC	Real Madrid CF
AS Roma	0.	0.	0.163	0.171	0.171	0.13	0.202	0.164
Panathinaikos FC	0.167	0.	0.137	0.141	0.142	0.109	0.166	0.137
FC Barcelona	0.282	0.248	0.	0.	0.	0.189	0.281	0.
Liverpool FC	0.	0.217	0.204	0.	0.216	0.16	0.	0.204
Manchester United FC	0.	0.216	0.205	0.216	0.	0.16	0.	0.203
FC Bayern Munchen	0.16	0.145	0.132	0.136	0.136	0.	0.16	0.131
FC Porto	0.198	0.175	0.	0.17	0.169	0.126	0.	0.161
Juventus	0.192	0.	0.16	0.165	0.166	0.126	0.191	0.

Table D.7. Season 2010

	FC Bayern Munchen	PFC CSKA Moskva	AC Milan	FC Porto	Olympique Lyonnais	FC Internazionale Milano	VfB Stuttgart	Olympiacos FC
FC Girondins de Bordeaux	0.	0.152	0.194	0.154	0.	0.194	0.153	0.153
Manchester United FC	0.131	0.	0.16	0.128	0.165	0.16	0.128	0.128
Real Madrid CF	0.135	0.132	0.	0.132	0.171	0.166	0.132	0.132
Chelsea FC	0.131	0.128	0.16	0.	0.164	0.16	0.129	0.128
ACF Fiorentina	0.206	0.199	0.	0.198	0.	0.	0.198	0.199
FC Barcelona	0.135	0.132	0.167	0.131	0.171	0.	0.132	0.132
Sevilla FC	0.131	0.128	0.16	0.128	0.165	0.16	0.	0.128
Arsenal FC	0.131	0.128	0.159	0.129	0.164	0.16	0.128	0.

Table D.8. Season 2011

	FC Internazionale Milano	Olympique Lyonnais	Valencia CF	FC Kobenhavn	AS Roma	Olympique de Marseille	AC Milan	Arsenal FC
Tottenham Hotspur FC	0.	0.153	0.227	0.156	0.151	0.158	0.155	0.
FC Schalke 04	0.121	0.	0.175	0.12	0.118	0.12	0.12	0.226
Manchester United FC	0.169	0.161	0.	0.168	0.163	0.17	0.168	0.
FC Barcelona	0.147	0.143	0.	0.	0.143	0.147	0.147	0.274
FC Bayern Munchen	0.121	0.118	0.175	0.119	0.	0.121	0.12	0.225
Chelsea FC	0.158	0.152	0.227	0.155	0.152	0.	0.156	0.
Real Madrid CF	0.147	0.142	0.	0.147	0.143	0.147	0.	0.274
FC Shakhtar Donetsk	0.137	0.13	0.196	0.134	0.131	0.137	0.134	0.

Table D.9. Season 2012

	SSC Napoli	PFC CSKA Moskva	FC Basel 1893	Olympique Lyonnais	Bayer 04 Leverkusen	Olympique de Marseille	FC Zenit	AC Milan
FC Bayern Munchen	0.	0.171	0.158	0.158	0.	0.158	0.157	0.198
FC Internazionale Milano	0.	0.	0.193	0.193	0.229	0.192	0.193	0.
SL Benfica	0.164	0.136	0.	0.129	0.154	0.129	0.129	0.159
Real Madrid CF	0.165	0.137	0.128	0.	0.154	0.128	0.129	0.16
Chelsea FC	0.17	0.139	0.131	0.132	0.	0.132	0.132	0.164
Arsenal FC	0.165	0.137	0.129	0.128	0.153	0.	0.129	0.16
APOEL FC	0.164	0.136	0.129	0.129	0.153	0.129	0.	0.16
FC Barcelona	0.172	0.144	0.132	0.132	0.157	0.132	0.132	0.

Table D.10. Season 2013

	FC Porto	Arsenal FC	AC Milan	Real Madrid CF	FC Shakhtar Donetsk	Valencia CF	Celtic FC	Galatasaray AS
Paris Saint-Germain	0.	0.134	0.144	0.183	0.117	0.184	0.121	0.116
FC Schalke 04	0.117	0.	0.146	0.187	0.119	0.187	0.124	0.12
Malaga CF	0.189	0.217	0.	0.	0.194	0.	0.209	0.192
Borussia Dortmund	0.122	0.142	0.155	0.	0.125	0.201	0.132	0.125
Juventus	0.135	0.156	0.	0.215	0.	0.216	0.141	0.137
FC Bayern Munchen	0.122	0.141	0.154	0.202	0.125	0.	0.132	0.124
FC Barcelona	0.183	0.21	0.235	0.	0.186	0.	0.	0.187
Manchester United FC	0.133	0.	0.166	0.212	0.135	0.213	0.141	0.

Table D.11. Season 2014

	Bayer 04 Leverkusen	Galatasaray AS	Olympiacos FC	Manchester City FC	FC Schalke 04	Arsenal FC	FC Zenit	AC Milan
Manchester United FC	0.	0.172	0.173	0.	0.31	0.	0.172	0.172
Real Madrid CF	0.173	0.	0.103	0.172	0.173	0.173	0.104	0.103
Paris Saint-Germain	0.172	0.103	0.	0.173	0.173	0.173	0.103	0.103
FC Bayern Munchen	0.	0.173	0.173	0.	0.	0.309	0.172	0.173
Chelsea FC	0.309	0.173	0.173	0.	0.	0.	0.173	0.172
Borussia Dortmund	0.	0.172	0.172	0.311	0.	0.	0.173	0.173
Club Atletico de Madrid	0.173	0.104	0.103	0.172	0.172	0.172	0.	0.104
FC Barcelona	0.173	0.103	0.103	0.172	0.173	0.173	0.103	0.

Table D.12. Season 2015

	Juventus	FC Basel 1893	Bayer 04 Leverkusen	Arsenal FC	Manchester City FC	Paris Saint-Germain	FC Schalke 04	FC Shakhtar Donetsk
Club Atletico de Madrid	0.	0.11	0.177	0.143	0.143	0.127	0.19	0.11
Real Madrid CF	0.11	0.	0.177	0.142	0.143	0.128	0.189	0.111
AS Monaco FC	0.135	0.135	0.	0.177	0.177	0.	0.24	0.135
Borussia Dortmund	0.18	0.181	0.	0.	0.25	0.209	0.	0.181
FC Bayern Munchen	0.18	0.181	0.	0.25	0.	0.209	0.	0.181
FC Barcelona	0.113	0.112	0.18	0.145	0.145	0.	0.193	0.112
Chelsea FC	0.172	0.171	0.288	0.	0.	0.198	0.	0.171
FC Porto	0.11	0.11	0.178	0.143	0.142	0.129	0.189	0.

Table D.13. Season 2016

	Paris Saint-Germain	PSV Eindhoven	SL Benfica	Juventus	AS Roma	Arsenal FC	FC Dynamo Kyiv	KAA Gent
Real Madrid CF	0.	0.129	0.129	0.131	0.129	0.193	0.158	0.131
VfL Wolfsburg	0.129	0.	0.128	0.132	0.129	0.192	0.158	0.132
Club Atletico de Madrid	0.129	0.128	0.	0.132	0.129	0.193	0.159	0.131
Manchester City FC	0.159	0.16	0.16	0.	0.16	0.	0.198	0.164
FC Barcelona	0.129	0.129	0.129	0.132	0.	0.193	0.158	0.131
FC Bayern Munchen	0.137	0.137	0.137	0.143	0.136	0.	0.169	0.14
Chelsea FC	0.164	0.164	0.164	0.172	0.165	0.	0.	0.171
FC Zenit	0.153	0.153	0.154	0.158	0.153	0.23	0.	0.

Table D.14. Season 2017

	Paris Saint-Germain	SL Benfica	Manchester City FC	FC Bayern Munchen	Bayer 04 Leverkusen	Real Madrid CF	FC Porto	Sevilla FC
Arsenal FC	0.	0.129	0.	0.165	0.158	0.216	0.133	0.199
SSC Napoli	0.127	0.	0.158	0.136	0.13	0.176	0.109	0.164
FC Barcelona	0.204	0.174	0.	0.231	0.212	0.	0.179	0.
Club Atletico de Madrid	0.197	0.169	0.252	0.	0.209	0.	0.173	0.
AS Monaco FC	0.	0.126	0.185	0.162	0.	0.208	0.127	0.192
Borussia Dortmund	0.188	0.161	0.236	0.	0.	0.	0.164	0.251
Leicester City FC	0.151	0.128	0.	0.162	0.154	0.21	0.	0.195
Juventus	0.133	0.113	0.168	0.145	0.137	0.19	0.114	0.

Table D.15. Season 2018

	FC Basel 1893	FC Bayern Munchen	Chelsea FC	Juventus	Sevilla FC	FC Shakhtar Donetsk	FC Porto	Real Madrid CF
Manchester United FC	0.	0.148	0.	0.183	0.183	0.155	0.148	0.182
Paris Saint-Germain	0.109	0.	0.294	0.128	0.128	0.108	0.105	0.128
AS Roma	0.159	0.151	0.	0.	0.189	0.16	0.152	0.189
FC Barcelona	0.149	0.144	0.413	0.	0.	0.15	0.144	0.
Liverpool FC	0.159	0.151	0.	0.189	0.	0.16	0.152	0.189
Manchester City FC	0.156	0.148	0.	0.183	0.184	0.	0.148	0.183
Besiktas JK	0.109	0.105	0.293	0.128	0.128	0.108	0.	0.129
Tottenham Hotspur FC	0.16	0.152	0.	0.189	0.189	0.159	0.151	0.

Table D.16. Season 2019

	Club Atletico de Madrid	Tottenham Hotspur FC	Liverpool FC	FC Schalke 04	AFC Ajax	Olympique Lyonnais	AS Roma	Manchester United FC
Borussia Dortmund	0.	0.175	0.175	0.	0.14	0.175	0.161	0.174
FC Barcelona	0.	0.	0.173	0.186	0.136	0.174	0.159	0.173
Paris Saint-Germain	0.186	0.174	0.	0.18	0.132	0.	0.155	0.173
FC Porto	0.163	0.146	0.147	0.	0.117	0.146	0.135	0.145
FC Bayern Munchen	0.184	0.166	0.167	0.	0.	0.166	0.152	0.165
Manchester City FC	0.282	0.	0.	0.273	0.208	0.	0.237	0.
Real Madrid CF	0.	0.171	0.17	0.183	0.135	0.171	0.	0.171
Juventus	0.185	0.168	0.169	0.179	0.131	0.168	0.	0.

Table D.17. Season 2020

	Real Madrid CF	Tottenham Hotspur FC	Atalanta BC	Club Atletico de Madrid	SSC Napoli	Borussia Dortmund	Olympique Lyonnais	Chelsea FC
Paris Saint-Germain	0.	0.172	0.146	0.184	0.145	0.17	0.	0.182
FC Bayern Munchen	0.181	0.	0.151	0.187	0.15	0.	0.143	0.189
Manchester City FC	0.216	0.	0.	0.224	0.185	0.206	0.17	0.
Juventus	0.211	0.207	0.	0.	0.	0.2	0.164	0.218
Liverpool FC	0.216	0.	0.184	0.224	0.	0.206	0.17	0.
FC Barcelona	0.	0.22	0.186	0.	0.186	0.	0.176	0.232
RB Leipzig	0.176	0.173	0.145	0.181	0.145	0.	0.	0.179
Valencia CF	0.	0.227	0.188	0.	0.189	0.218	0.177	0.

Table D.18. Season 2021

	Club Atletico de Madrid	VfL Borussia Monchengladbach	FC Porto	Atalanta BC	Sevilla FC	SS Lazio	FC Barcelona	RB Leipzig
FC Bayern Munchen	0.	0.	0.164	0.193	0.211	0.209	0.223	0.
Real Madrid CF	0.	0.	0.204	0.237	0.	0.254	0.	0.305
Manchester City FC	0.14	0.181	0.	0.124	0.131	0.131	0.136	0.157
Liverpool FC	0.143	0.183	0.107	0.	0.133	0.134	0.14	0.16
Chelsea FC	0.145	0.186	0.107	0.126	0.	0.134	0.141	0.161
Borussia Dortmund	0.228	0.	0.162	0.191	0.204	0.	0.215	0.
Juventus	0.196	0.256	0.147	0.	0.183	0.	0.	0.217
Paris Saint-Germain	0.148	0.194	0.11	0.129	0.137	0.138	0.144	0.

Table D.19. Season 2022

	Paris Saint-Germain	Club Atletico de Madrid	Sporting Clube de Portugal	FC Internazionale Milano	SL Benfica	Villarreal CF	FC Red Bull Salzburg	Chelsea FC
Manchester City FC	0.	0.185	0.148	0.185	0.148	0.184	0.151	0.
Liverpool FC	0.179	0.	0.149	0.187	0.148	0.187	0.15	0.
AFC Ajax	0.135	0.14	0.	0.139	0.113	0.14	0.115	0.217
Real Madrid CF	0.194	0.	0.163	0.	0.163	0.	0.166	0.314
FC Bayern Munchen	0.136	0.141	0.113	0.14	0.	0.14	0.114	0.217
Manchester United FC	0.178	0.186	0.148	0.188	0.148	0.	0.151	0.
LOSC Lille	0.	0.162	0.131	0.162	0.131	0.161	0.	0.252
Juventus	0.178	0.187	0.149	0.	0.149	0.187	0.151	0.

Appendix E. Near-optimality of the UEFA procedure

E.1. **Fairness objective for an eight-by-eight assignment.**

E.1.1. *Small-dimension analysis.*

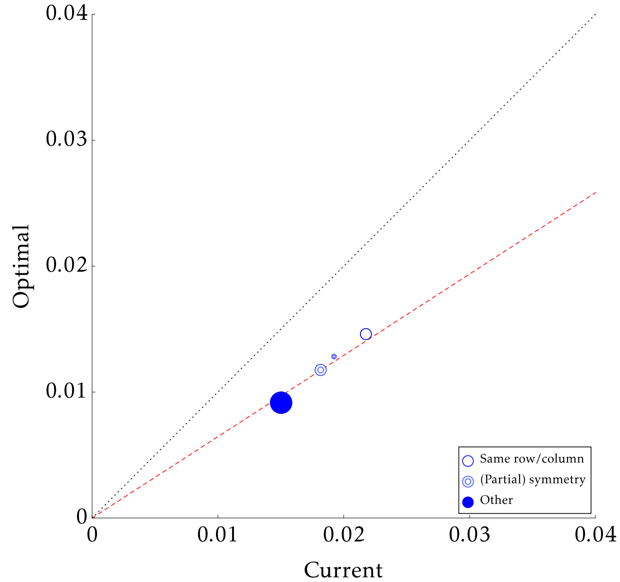


Figure E.1.1.1. Number of constraints: $C = 2$

Figure details: Red dashed line indicates fitted linear relationship.

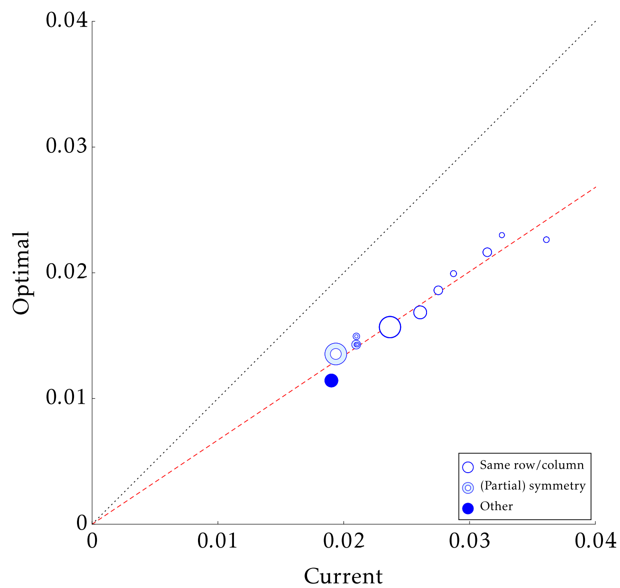


Figure E.1.1.2. Number of constraints: $C = 3$

Figure details: Red dashed line indicates fitted linear relationship.

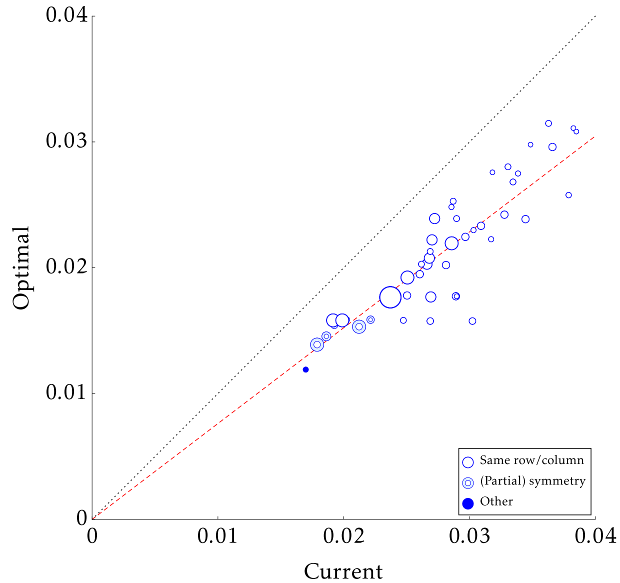


FIGURE E.1.1.3. Number of constraints: $C = 4$

Figure details: Red dashed line indicates fitted linear relationship.

E.1.2. *Monte-Carlo simulation exercise: Independent Simulations.*

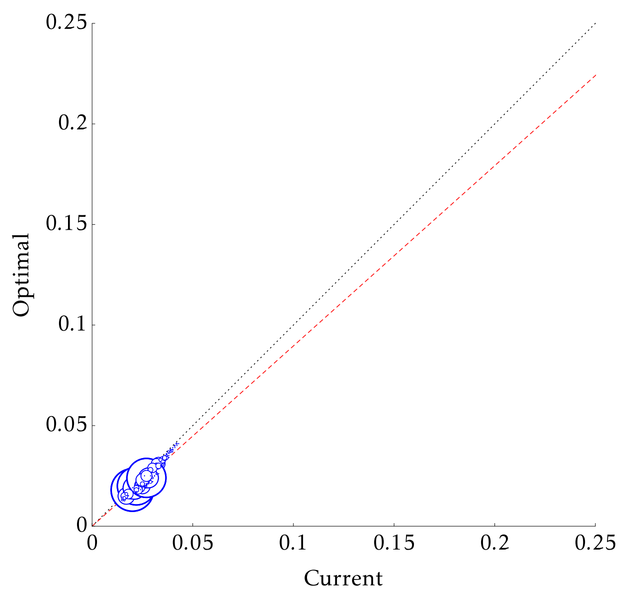


FIGURE E.1.2.1. Number of constraints: $C = 5$

Figure details: Red dashed line indicates fitted linear relationship.

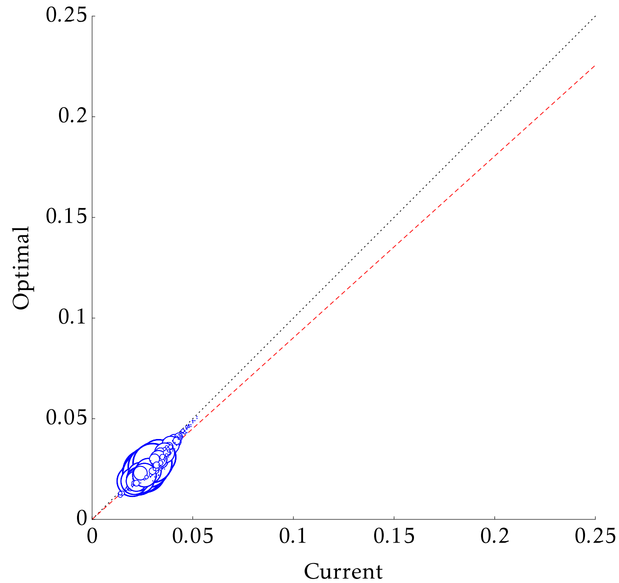


FIGURE E.1.2.2. Number of constraints: $C = 6$

Figure details: Red dashed line indicates fitted linear relationship.

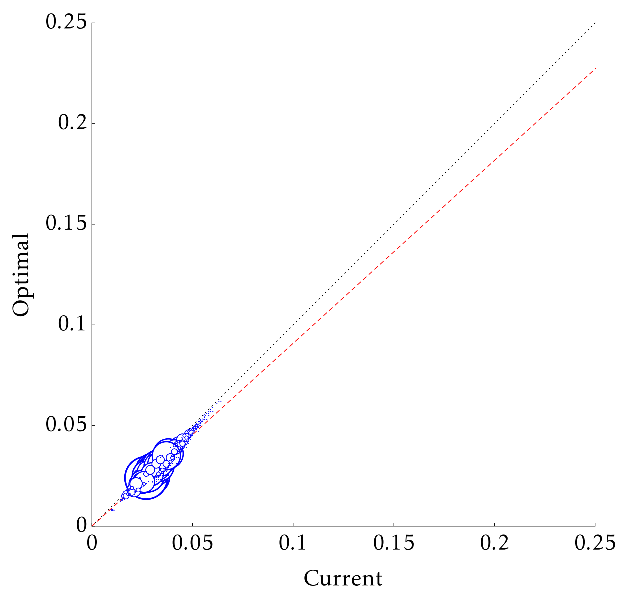


FIGURE E.1.2.3. Fairness objective under independent constraint structures: Optimal versus current, $C = 7$

Figure details: Red dashed line indicates fitted linear relationship.

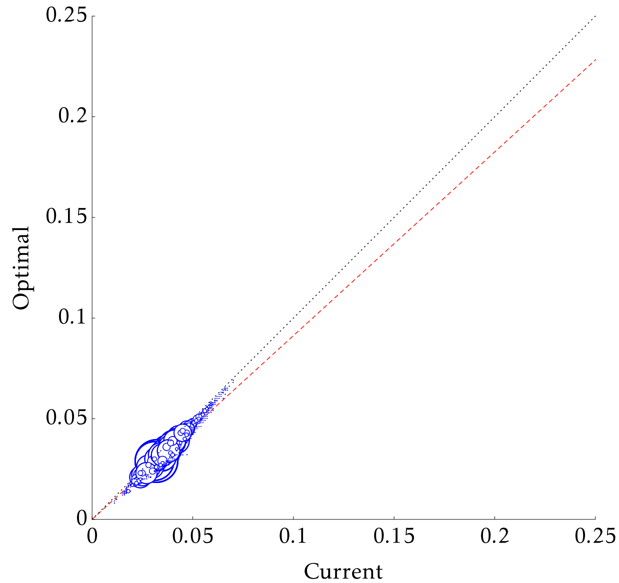


Figure E.1.2.4. Number of constraints: $C = 8$

Figure details: Red dashed line indicates fitted linear relationship.

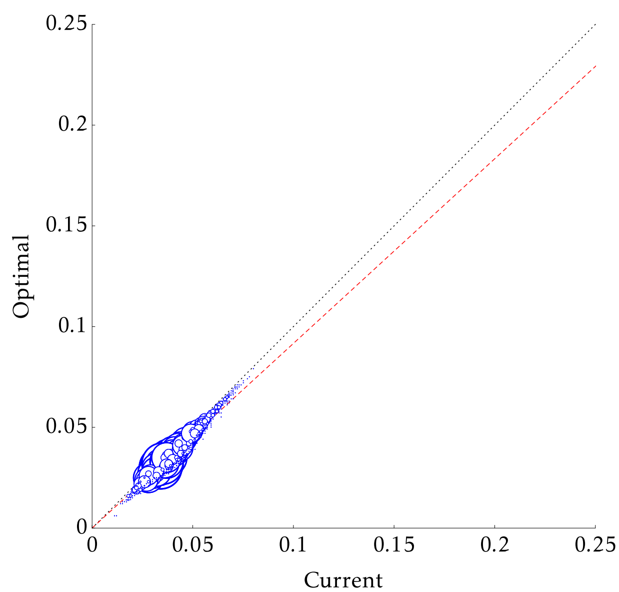


Figure E.1.2.5. Number of constraints: $C = 9$

Figure details: Red dashed line indicates fitted linear relationship.

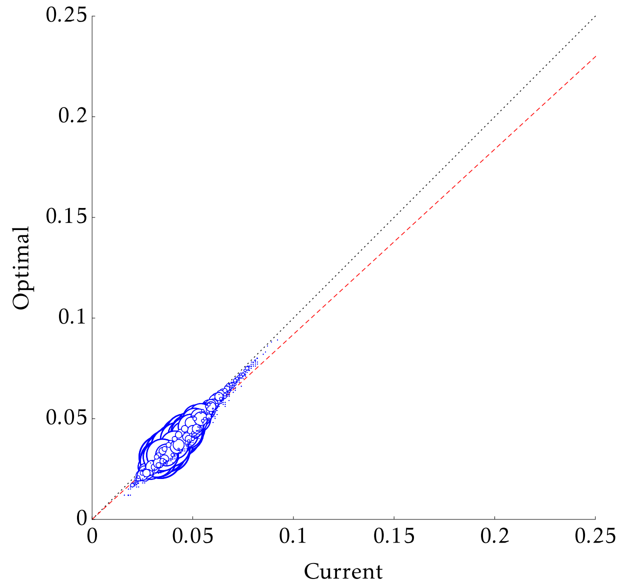


FIGURE E.1.2.6. Number of constraints: $C = 10$

Figure details: Red dashed line indicates fitted linear relationship.

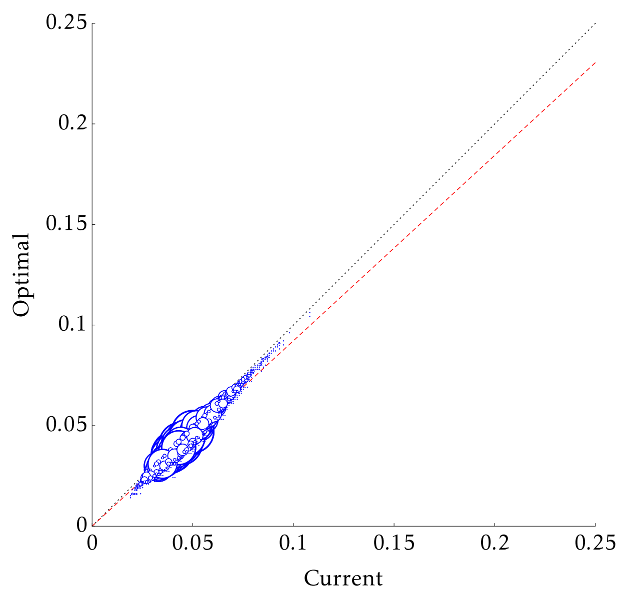


FIGURE E.1.2.7. Number of constraints: $C = 11$

Figure details: Red dashed line indicates fitted linear relationship.

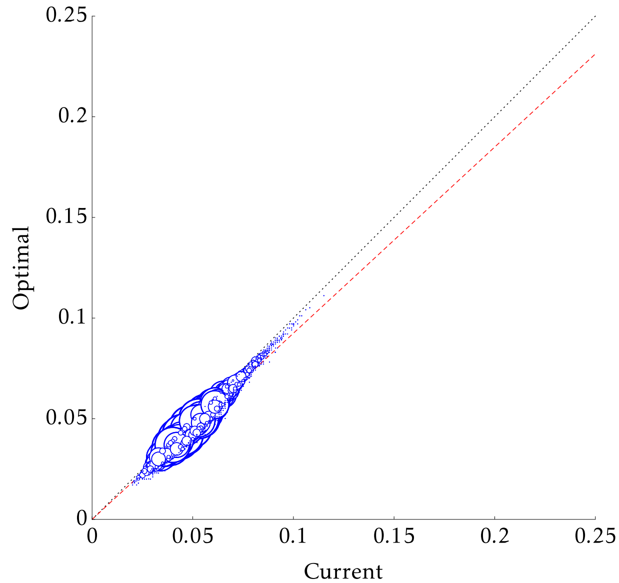


FIGURE E.1.2.8. Number of constraints: $C = 12$

Figure details: Red dashed line indicates fitted linear relationship.

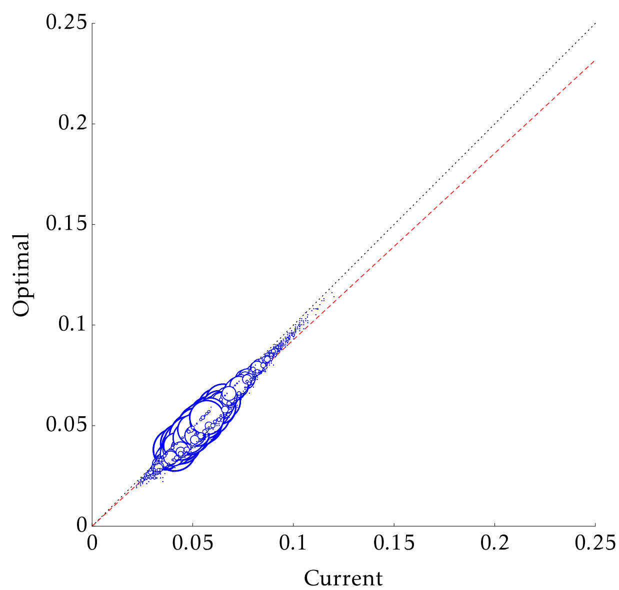


FIGURE E.1.2.9. Number of constraints: $C = 13$

Figure details: Red dashed line indicates fitted linear relationship.

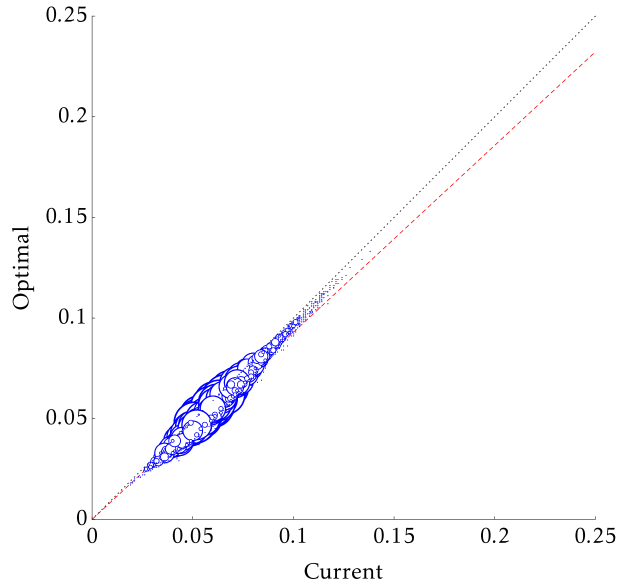


Figure E.1.2.10. Number of constraints: $C = 14$

Figure details: Red dashed line indicates fitted linear relationship.

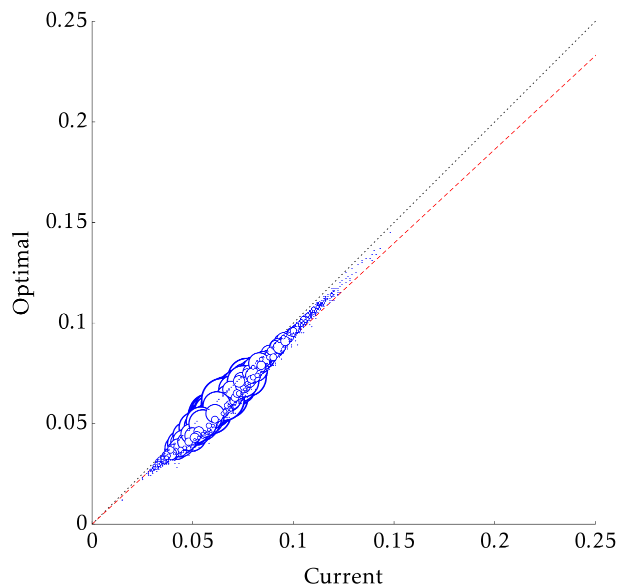


Figure E.1.2.11. Number of constraints: $C = 15$

Figure details: Red dashed line indicates fitted linear relationship.

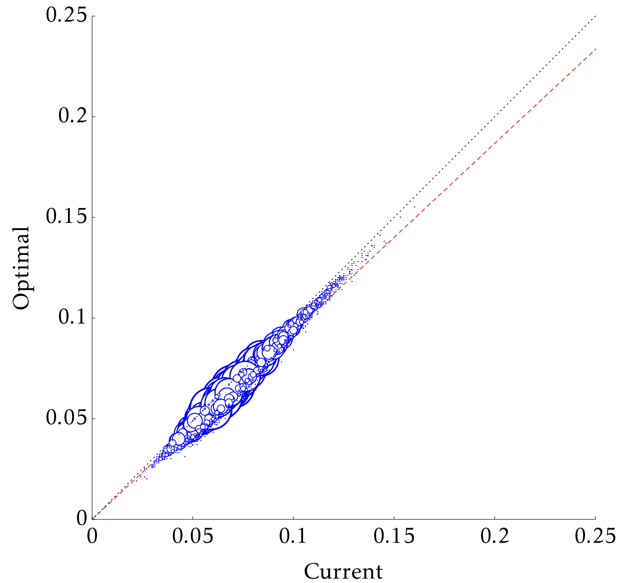


Figure E.1.2.12. Number of constraints: $C = 16$

Figure details: Red dashed line indicates fitted linear relationship.

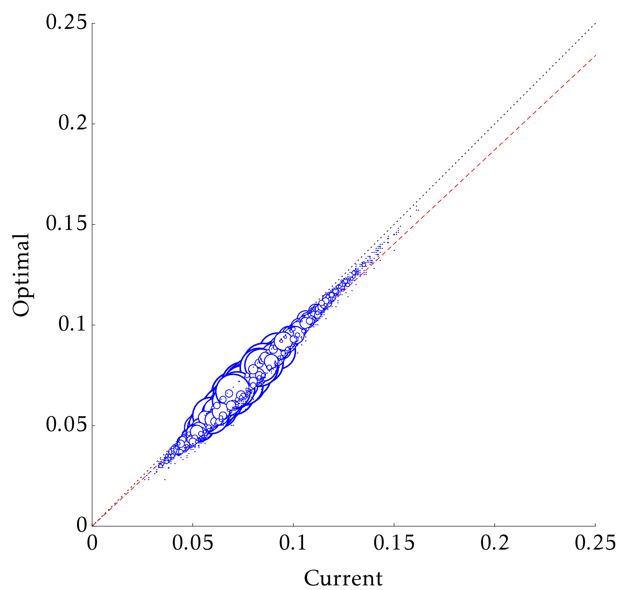


Figure E.1.2.13. Number of constraints: $C = 17$

Figure details: Red dashed line indicates fitted linear relationship.

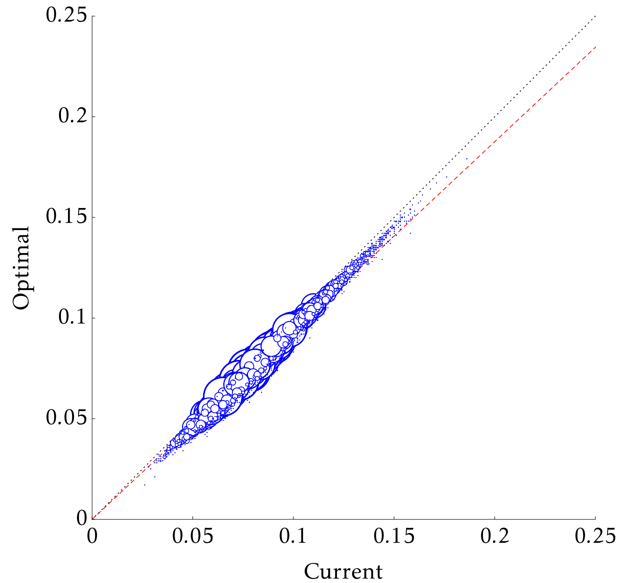


FIGURE E.1.2.14. Number of constraints: $C = 18$

Figure details: Red dashed line indicates fitted linear relationship.

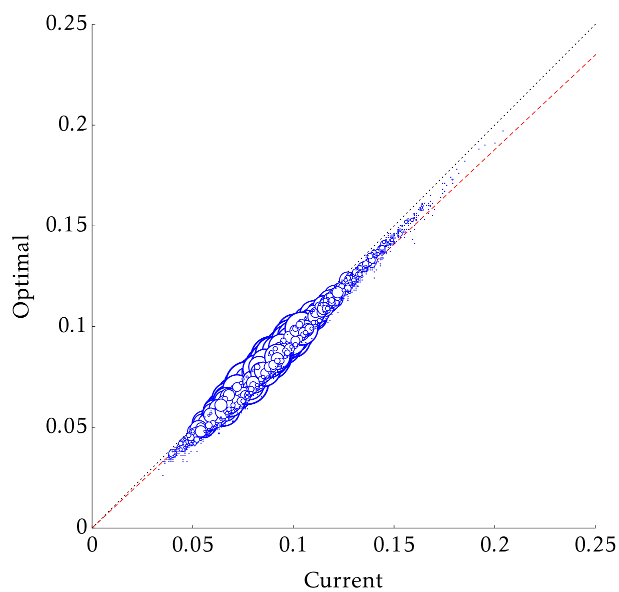


FIGURE E.1.2.15. Number of constraints: $C = 19$

Figure details: Red dashed line indicates fitted linear relationship.

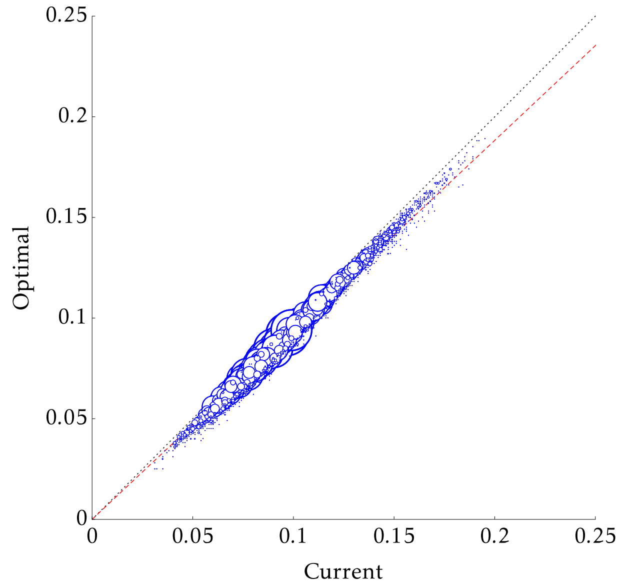


FIGURE E.1.2.16. Number of constraints: $C = 20$

Figure details: Red dashed line indicates fitted linear relationship.

E.1.3. *Monte-Carlo simulation exercise: Correlated draws.*
Draw procedure of C-correlated constraints: Let $\mathcal{U} = \{(ij) : i, j = 1, ..., K, i \neq j\}$, be a set of all off-diagonal elements of $K \times K$ matrix. Let $\mathcal{B}$ denote a set of sampling weights of elements in $\mathcal{U}$. Let $\mathcal{H}_A$ be the set of C drawn constraints Let m_i for all $i = 1, ..., K$ denote the cumulative number of times element $(i\cdot)$ has been drawn. Let m_j for all $j = 1, ..., K$ denote the cumulative number of times element $(\cdot j)$ has been drawn.

Algorithm. ***Initialization***: *Set* $H_{A,0} = \emptyset$, $\mathcal{B} = \mathcal{B}_0 = 1$, $m_{i,0} = m_{j,0} = 0$.
Step-*c*: *(for* $c = 1$ *to* C*)*

(1) *Chose* $(i_c j_c)$ *through a weighted draw over* $\mathcal{U}$ *with weights in* $\mathcal{B}_c$.
(2) *Define the set of currently drawn constraints:* $H_{A,c} = H_{A,c-1} \cup (i_c j_c)$.
(3) *Update the values of multiplicity factors by the latest draw:*
$\forall i = 1, ..., K : m_{i,c} = m_{i,c-1} + \mathbf{1}_{i==i_c}$
$\forall j = 1, ..., K : m_{j,c} = m_{j,c-1} + \mathbf{1}_{j==j_c}$
(4) *Define the set of current sampling weights:* $\mathcal{B}_c = \{b_{ij,c}\}$ *where*
$\forall (ij) \in H_{A,c} : b_{ij,c} = 0$
$\forall (ij) \in \mathcal{U} / \ H_{A,c} : b_{ij,c} = b^*_{max\{m_{i,c}, mj,c\}}$

For positively correlated constraints we set $[b^*_1, b^*_2, b^*_3, b^*_4, b^*_5] = [3, 6, 9, 12, 15]$. *For negatively correlated constraints we set* $[b^*_1, b^*_2, b^*_3, b^*_4, b^*_5] = [1/3, 1/6, 1/9, 1/12, 1/15]$.

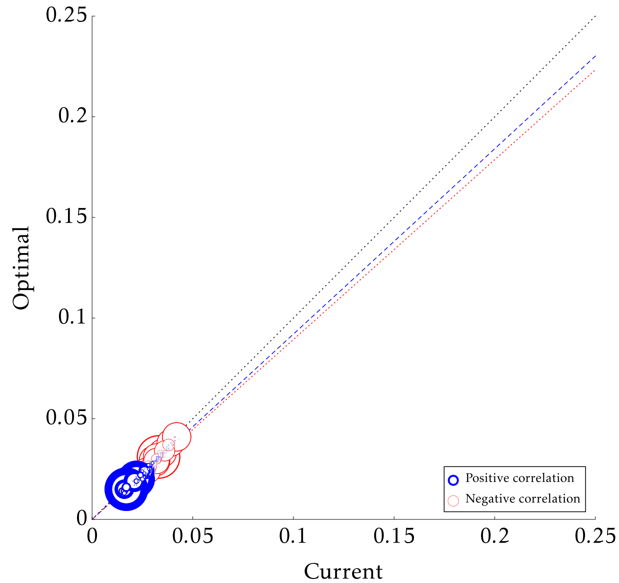


Figure E.1.3.1. Number of constraints: $C = 5$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

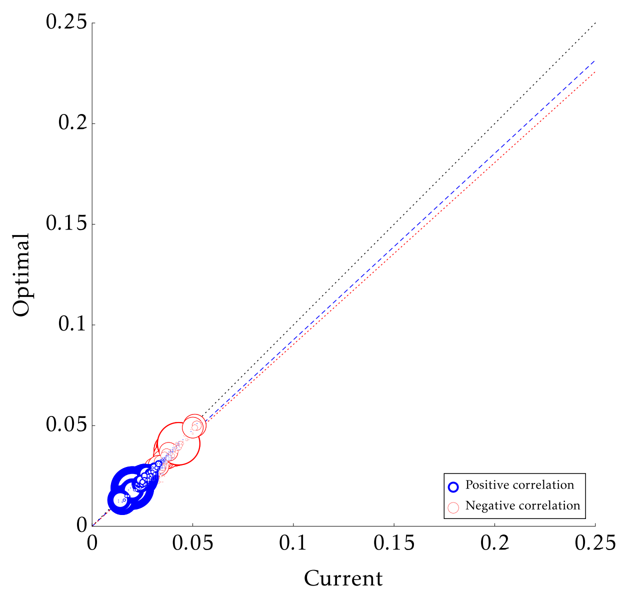


Figure E.1.3.2. Number of constraints: $C = 6$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

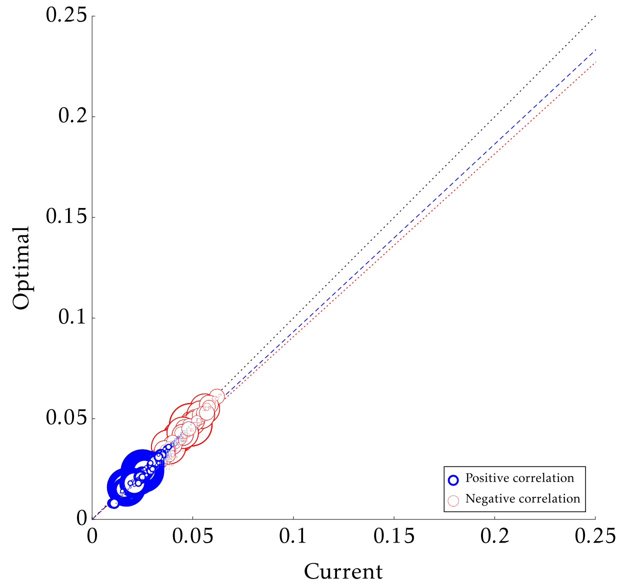


Figure E.1.3.3. Number of constraints: $C = 7$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

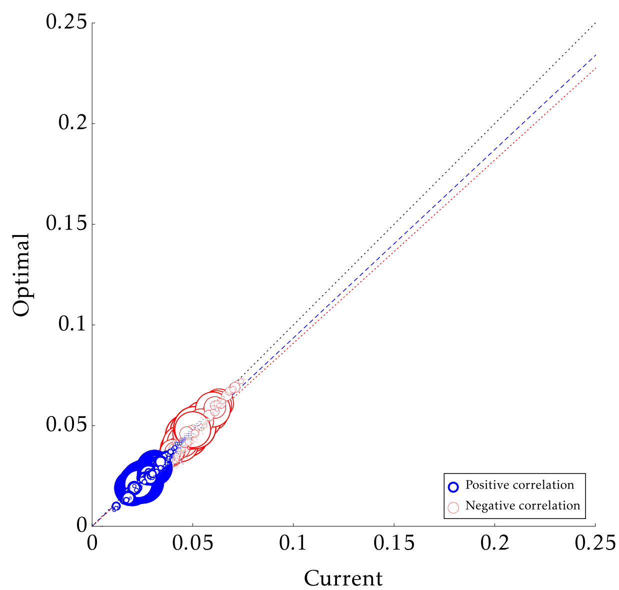


Figure E.1.3.4. Number of constraints: $C = 8$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

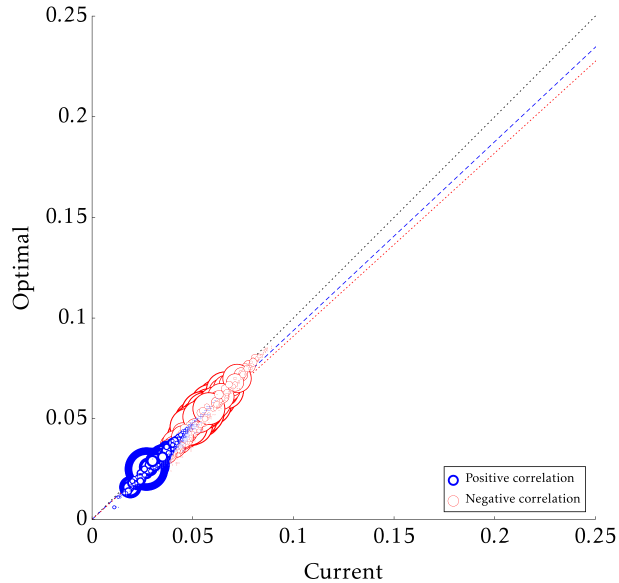


Figure E.1.3.5. Number of constraints: $C = 9$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

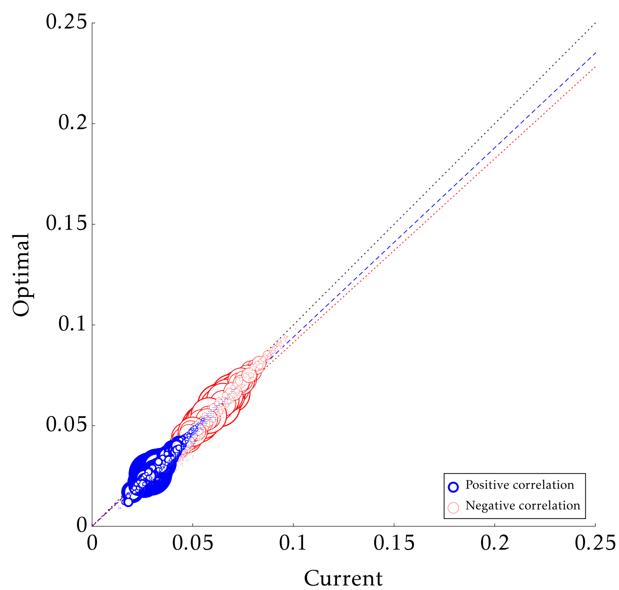


Figure E.1.3.6. Number of constraints:, $C = 10$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

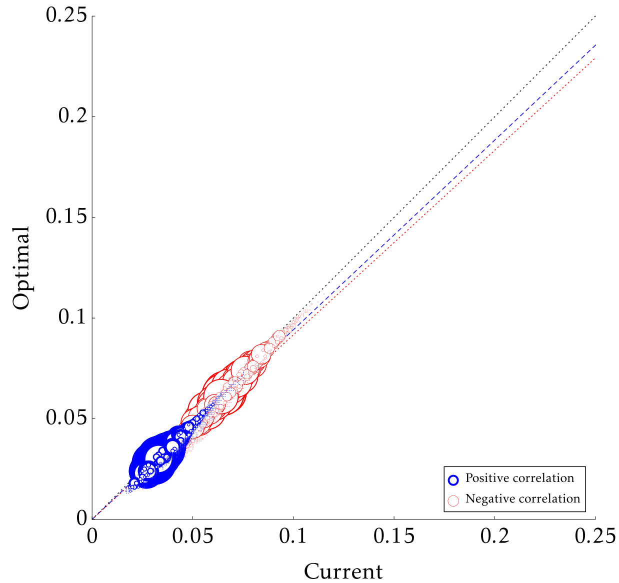


Figure E.1.3.7. Number of constraints: $C = 11$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

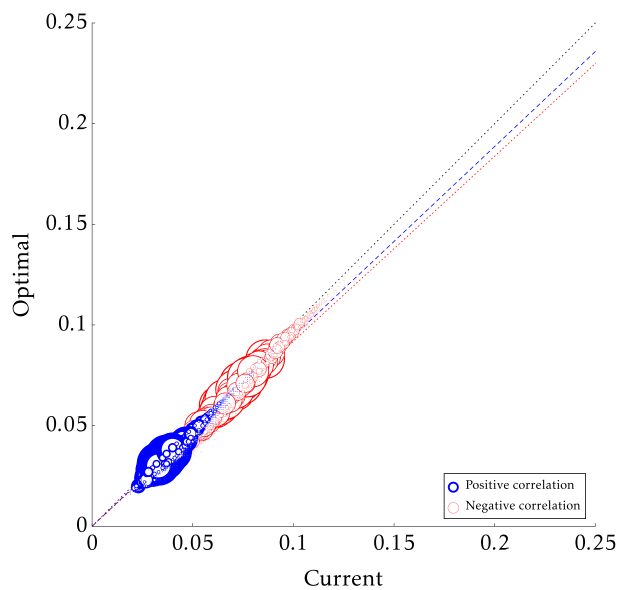


Figure E.1.3.8. Number of constraints: $C = 12$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

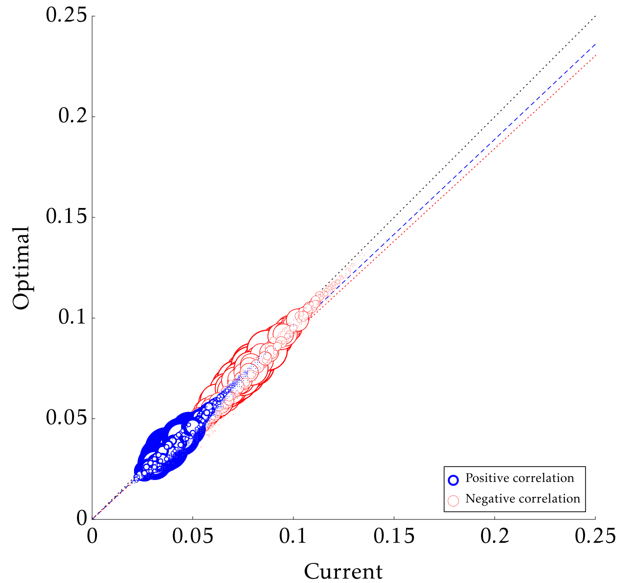


FIGURE E.1.3.9. Number of constraints: $C = 13$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

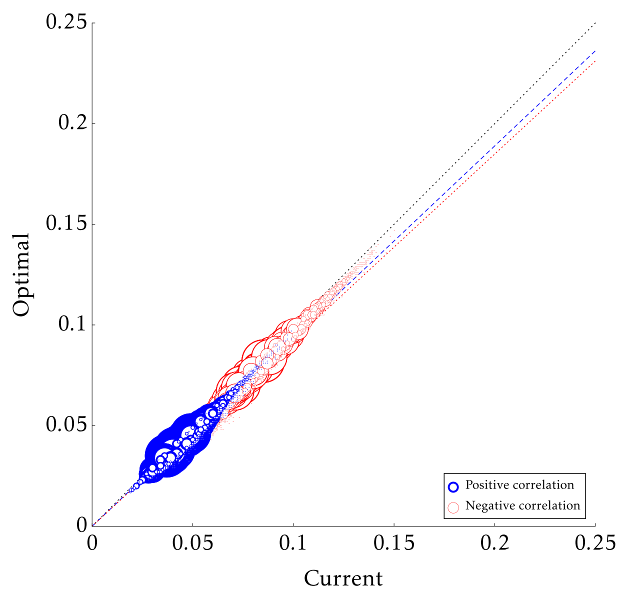


FIGURE E.1.3.10. Number of constraints: $C = 14$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

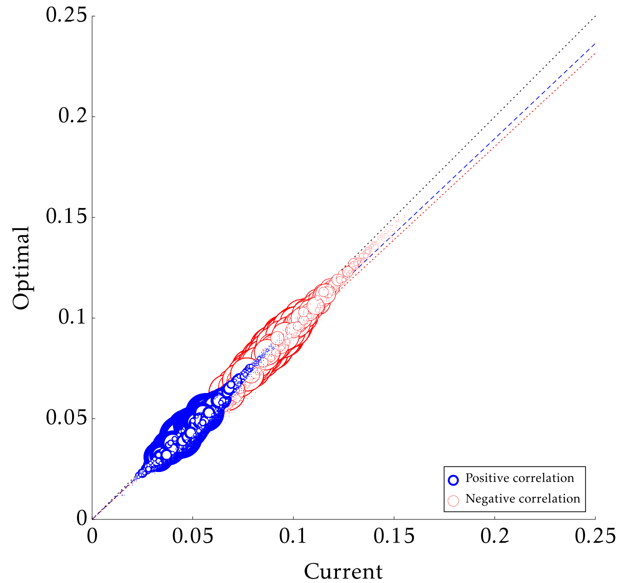


FIGURE E.1.3.11. Number of constraints: $C = 15$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

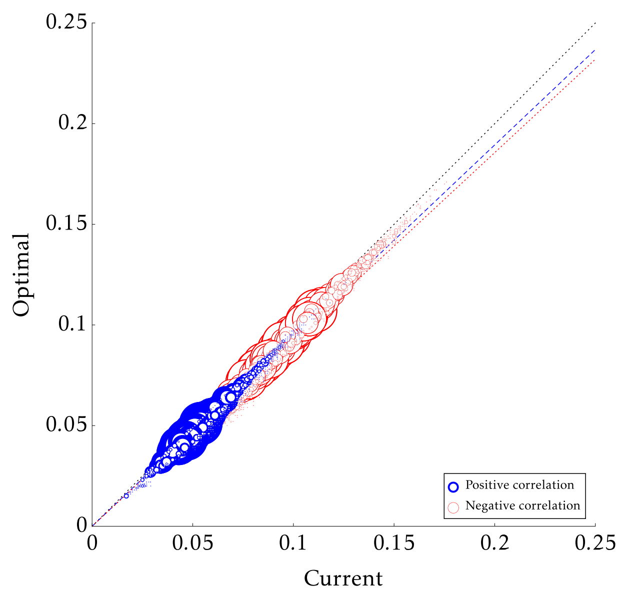


FIGURE E.1.3.12. Number of constraints: $C = 16$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

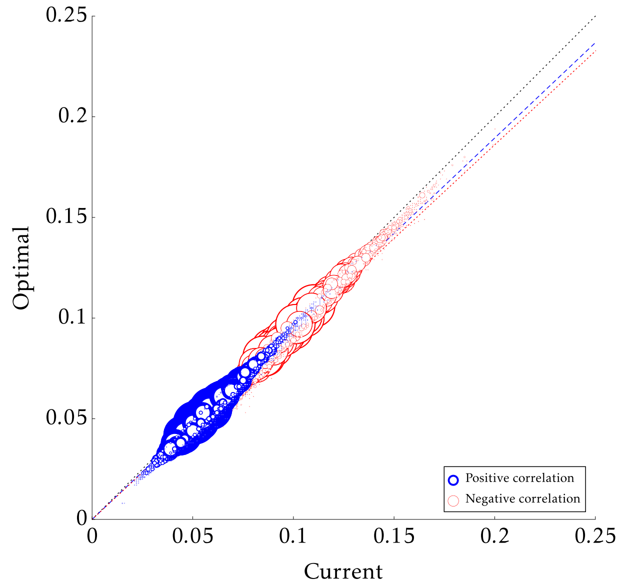


FIGURE E.1.3.13. Number of constraints: $C = 17$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

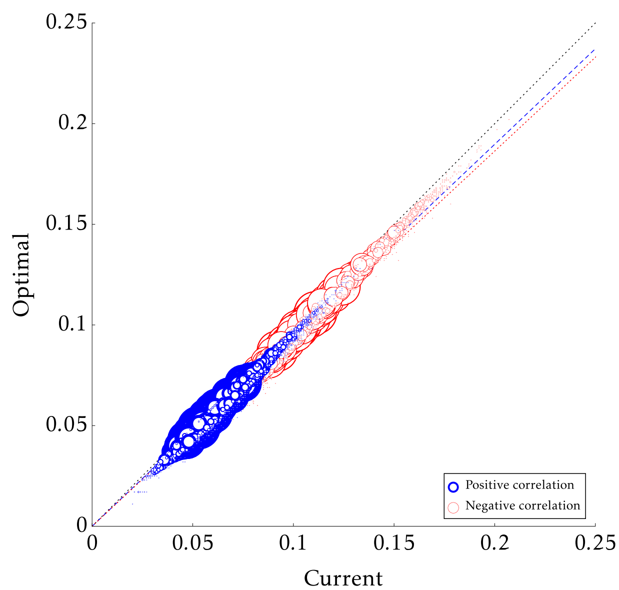


FIGURE E.1.3.14. Number of constraints: $C = 18$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

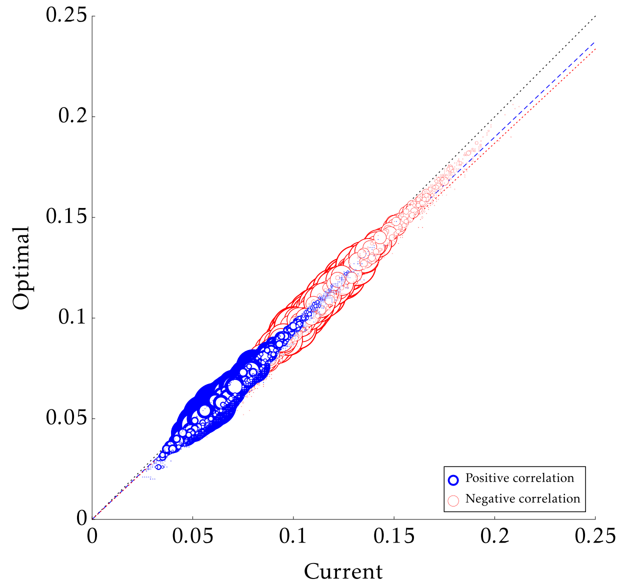


FIGURE E.1.3.15. Number of constraints: $C = 19$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

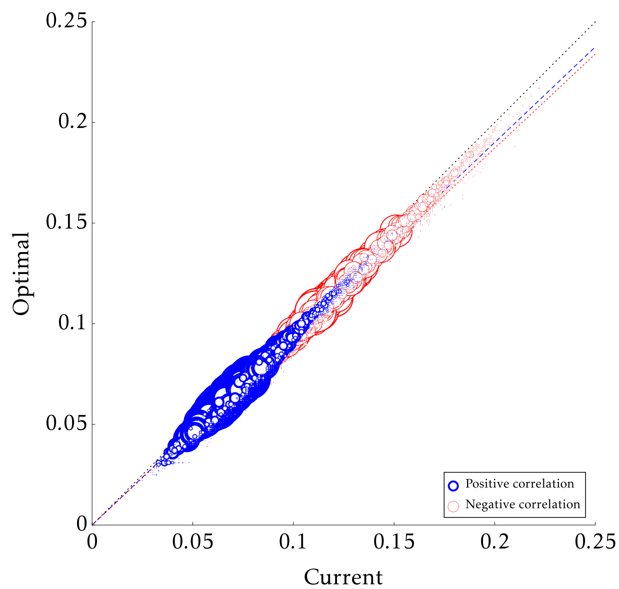
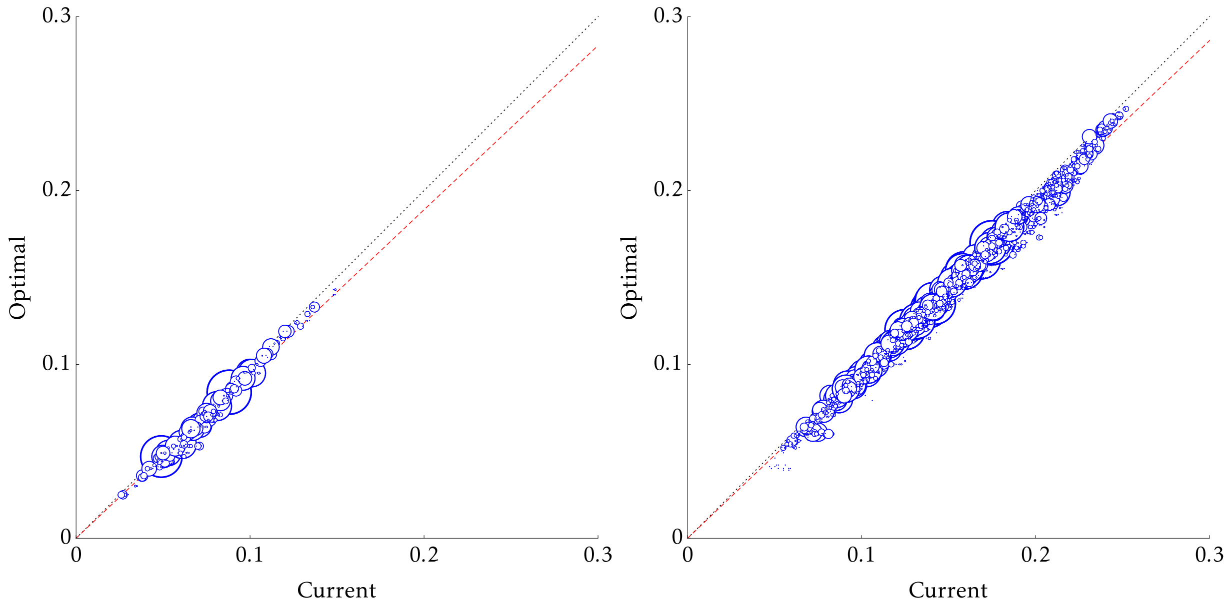


FIGURE E.1.3.16. Number of constraints: $C = 20$

Figure details: Red dotted line indicates fitted linear relationship for negatively correlated constraint structures. Blue dashed line indicates fitted linear relationship for positively correlated constraint structures.

E.2. **Fairness objective for other assignments.**

E.2.1. *Six-by-six assignment.*

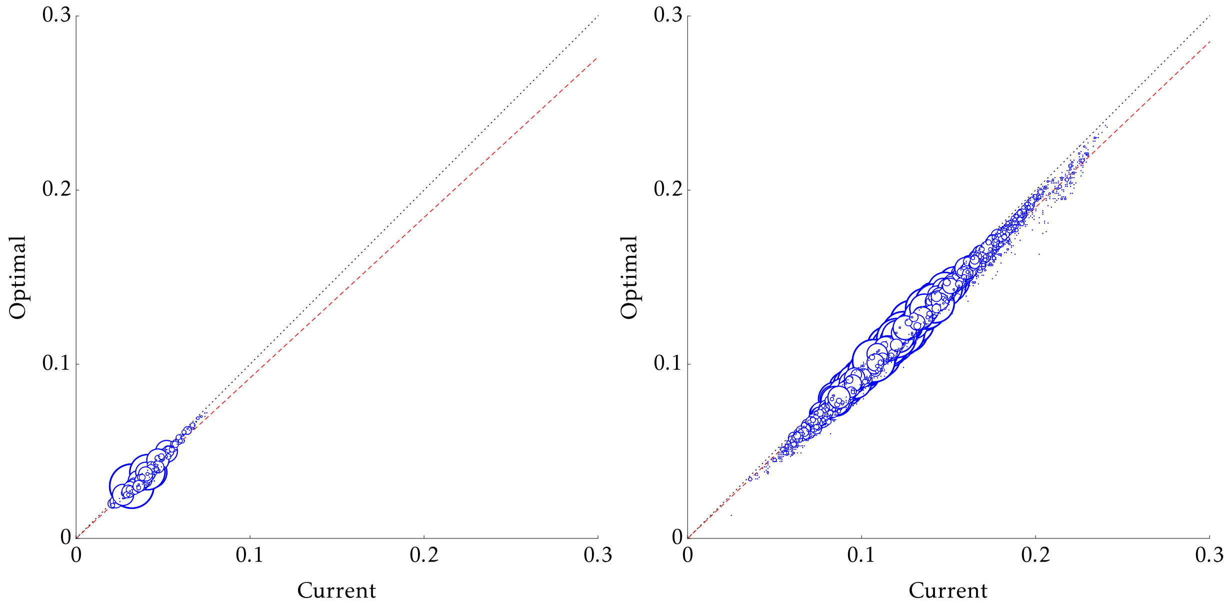


(A) Number of constraints: $C = 5$

(B) Number of constraints: $C = 10$

FIGURE E.2.1.1. Red dashed line indicates fitted linear relationship

E.2.2. *Seven-by-seven assignment.*



(A) Number of constraints: $C = 5$

(B) Number of constraints: $C = 15$

FIGURE E.2.2.1. Red dashed line indicates fitted linear relationship

References

Budish, Eric, Yeon-Koo Che, Fuhito Kojima, and Paul Milgrom, "Designing Random Allocation Mechanisms: Theory and Applications," *American Economic Review*, April 2013, *103* (2), 585–623.

Dixon, Mark J and Stuart G Coles, "Modelling association football scores and inefficiencies in the football betting market," *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 1997, *46* (2), 265–80.

Karlis, Dimitris and Ioannis Ntzoufras, "Analysis of sports data by using bivariate Poisson models," *The Statistician*, 2003, *52* (3), 381–93.

Maher, Michael J, "Modelling association football scores," *Statistica Neerlandica*, 1982, *36* (3), 109–18.